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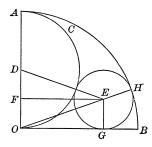
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Whence,

$$(2R-r)^2 - r^2 = (R+r)^2 - (R-r)^2.$$
  
 $r = \frac{R}{2}$ , or  $EG = \frac{1}{2}OD$ .



# II. SOLUTION BY HORACE OLSON, Chicago, Illinois.

Let x and y be the coördinates of the center of the required circle, referred to OB and OA as the positive directions on the x and y axes, respectively. Then by the conditions of the problem:

$$y = r - \sqrt{x^2 + y^2}, \quad y = \sqrt{\left(\frac{r}{2} - y\right)^2 + x^2} - \frac{r}{2}; \quad r = OA;$$

whence

$$r^2 - 2ry - x^2 = 0$$
, and  $2ry - x^2 = 0$ .

Subtracting, we have  $r^2 - 4ry = 0$ ; whence  $y = \frac{1}{4}r$  and  $x = \pm \frac{1}{2}\sqrt{2}r$ . The value  $x = + (r/2)\sqrt{2}$  is evidently the only one applying to this problem. The radius of the required circle is therefore  $\frac{1}{4}r$ .

A solution similar to the second above was received from Elijah Swift.

435 (Calculus). Proposed by B. F. FINKEL, Drury College.

Show that

$$\int_0^\infty e^{-x^2 - (a^2|x^2)} dx = \frac{\sqrt{\pi}}{2e^{2a}}$$

by a transformation rather than by the usual method of differentiating under the sign of integration, as, for example, in Byerly's *Integral Calculus*, pages 106–107.

### NOTE BY WILLIAM HOOVER, Columbus, Ohio.

On this interesting equation, reference may be made to an article by William Walton in the Quarterly Journal of Mathematics, Vol. XII, p. 181, On the Evaluation of a Pair of Definite Integrals, a few lines opening it being as follows:

"The evaluation of the two definite integrals

$$\int_0^\infty e^{-\left[x^2+\left(c^2/x^2\right)\right]\cos\alpha}\cdot\cos\left\{\left(x^2+\frac{c^2}{x^2}\right)\sin\alpha\right\}dx,$$

$$\int_0^\infty e^{-\left[x^2+\left(c^2/x^2\right)\right]\cos\alpha}\cdot\sin\left\{\left(x^2+\frac{c^2}{x^2}\right)\sin\alpha\right\}dx$$

is effected in textbooks on the integral calculus by the substitution of an impossible expression for k in the known relation

$$\int_0^\infty e^{-[x^2+(c^2/x^2)]^k} dx = \frac{\pi^{1/2}}{2k^{1/2}} \cdot e^{-2ck}.$$

"As such a method of arriving at the evaluation of definite integrals is regarded by eminent writers on the subject as suggestive rather than demonstrative, I think that the following evaluations of the two integrals in question may be of some interest to students."

He proceeds with the demonstrations, occupying two and one half pages of the *Quarterly*, avoiding the methods he criticizes and that by differentiation referred to in Mr. Finkel's problem, closing with the words "the results here obtained coincide, it may be remarked, with those given in Todhunter's *Int. Cal.*"

To solve Professor Finkel's problem we need only put  $\alpha = 0$  in the first of the integrals mentioned by Walton.

#### 437 (Calculus). Proposed by LEIGH PAGE, Yale University.

Integrate

$$\int_{-\infty}^{+\infty} \frac{\sin^2 x}{x^2} \, dx$$

without the use of the gamma functions

# I. Solution by Oscar S. Adams, Coast and Geodetic Survey, Washington, D. C.

By direct integration, we have

$$\int_0^\infty e^{-ax} dx \, = \frac{1}{a} \, .$$

Let  $a = \alpha - i\beta$ . Then

$$\int_0^\infty e^{-ax+i\beta x}dx = \frac{1}{\alpha - i\beta} = \frac{\alpha + i\beta}{\alpha^2 + \beta^2},$$

or

$$\int_0^\infty e^{-ax}(\cos\beta x + i\sin\beta x)dx = \frac{\alpha + i\beta}{\alpha^2 + \beta^2}.$$

By equating real and imaginary parts we obtain the two definite integrals

$$\int_0^\infty e^{-ax}\cos\beta x dx = \frac{\alpha}{\alpha^2 + \beta^2}, \quad \text{and} \quad \int_0^\infty e^{-ax}\sin\beta x dx = \frac{\beta}{\alpha^2 + \beta^2}.$$

Since the first of these integrals is a uniform function of  $\beta$ , we have the relation

$$\int_0^\beta d\beta \int_0^\infty e^{-\alpha x} \cos \beta x dx = \int_0^\infty dx \int_0^\beta e^{-\alpha x} \cos \beta x d\beta = \int_0^\beta \frac{\alpha d\beta}{\alpha^2 + \beta^2};$$

or

$$\int_0^\infty e^{-ax} \frac{\sin \beta x}{x} dx = \tan^{-1} \frac{\beta}{\alpha}.$$

This integral is also a uniform function of  $\beta$  in the region  $+\frac{\pi}{2} \ge \tan^{-1}\frac{\beta}{\alpha} \ge -\frac{\pi}{2}$ . Hence, in this domain, we have

 $\int_0^\infty dx \int_0^2 e^{-ax} \frac{\sin \beta x}{x} d\beta = \int_0^2 \tan^{-1} \frac{\beta}{\alpha} d\beta$ 

 $\mathbf{or}$ 

$$\int_0^\infty \frac{e^{-\alpha x}(1-\cos 2x)}{x^2} \, dx = 2 \, \tan^{-1} \frac{2}{\alpha} - \frac{\alpha}{2} \log \, (\alpha^2 + 4).$$

This is a uniform function of  $\alpha$ . Hence,  $\alpha$  can converge to zero. This gives

$$\int_0^\infty \frac{2\sin^2 x}{x^2} dx = 2 \cdot \frac{\pi}{2} = \pi, \quad \text{or} \quad \int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{1}{2}\pi.$$

Since  $\sin^2 x/x^2$  is an even function of x, we have

$$\int_{-\infty}^{\infty} \frac{\sin^2 x}{r^2} dx = 2 \int_0^{\infty} \frac{\sin^2 x}{r^2} dx = \pi.$$

### II. SOLUTION BY G. PAASWELL, N. Y. City.

Since  $\sin^2 x/x^2$  is an even function of x we have